

Cumulative Recalculation Reviews - Teacher Answer Guide

Purpose and use

Use these answers as routing evidence, not only as marks. A single error is a hypothesis. Confirm the pattern with a second item before assigning reteaching. Equivalent reasoning and representations are acceptable when the relationship, units, and check are correct.

Answer-sheet format

Each entry provides the expected answer, a plausible wrong response, the likely misconception, and the smallest next move. The misconception is a hypothesis until confirmed by another piece of evidence.

Part 1 - Foundations of Proportional Thinking

1. $k = 3$. The quotient y/x is 3 for every nonzero pair and $(0,0)$ satisfies the zero case.

Likely wrong response: uses only the constant difference.

Likely misconception: additive change substituted for multiplicative evidence.

Next move: require two quotients and the zero case.

2. Not proportional. The rule is $y = 2x + 4$, so the output is 4 when the input is 0 and the quotient changes.

Likely wrong response: calls it proportional because every output rises by 4.

Likely misconception: constant difference confused with constant ratio.

Next move: compare y/x at $x = 2$ and $x = 4$.

3. $3/4$ of a whole; 3 divided by 4; $(3/4) \times 20 = 15$.

Likely wrong response: draws unequal parts or treats $3/4$ as 3×4 .

Likely misconception: denominator meaning is insecure.

Next move: rebuild four equal parts and select three.

4. Less than 30 because $5/6$ is less than 1; exact answer 25.

Likely wrong response: answers greater than 30.

Likely misconception: magnitude of a fraction multiplier is not checked.

Next move: compare the multiplier with 1 before calculating.

5. Dividing by 2 halves $3/4$, giving $3/8$. Dividing by $1/2$ asks how many halves fit in $3/4$, giving $3/2$.

Likely wrong response: treats both divisors as producing a smaller answer.

Likely misconception: division by a fraction is not interpreted.

Next move: model grouping with half-unit strips.

6. Answers vary. Both situations must preserve the same multiplier while naming different quantities and units.

Likely wrong response: changes both the units and multiplier.

Likely misconception: surface context is mistaken for mathematical structure.

Next move: write the equation for each situation and compare constants.

7. The claim is false. $\frac{2}{3} = \frac{4}{6}$, not $\frac{5}{6}$; numerator and denominator must be scaled by the same factor.

Likely wrong response: accepts equal additive changes.

Likely misconception: equivalence is treated additively.

Next move: identify one scale factor that acts on both terms.

8. Answers vary. The response must name a representation and explain what relationship it made visible.

Likely wrong response: names a representation without justification.

Likely misconception: reflection lacks evidence.

Next move: ask what could be seen or checked in that representation.

Recalculation decision

Route to the narrowest confirmed gap. Record whether the evidence points to relationship classification, representation, operation, computation, units or base, interpretation, or checking. Do not route from one isolated miss.

Part 2 - Fractions, Decimals, and Percents

1. $\frac{3}{8} = 0.375 = 37.5\%$; all three locate the same point between 0 and 1.

Likely wrong response: places 37.5% at 3.75.

Likely misconception: percent scale is misread.

Next move: anchor 0%, 50%, and 100% on the number line.

2. 35% of 80 = 28. A benchmark estimate is near 30 because 37.5% of 80 is 30.

Likely wrong response: multiplies 35 x 80.

Likely misconception: percent is not converted to 0.35.

Next move: write 35% as 0.35 before calculating.

3. Part 24, percent 30%, whole 80; 24 divided by 0.30 = 80.

Likely wrong response: finds 7.2.

Likely misconception: missing whole treated as missing part.

Next move: label part, percent, and whole before choosing the operation.

4. $\frac{18}{24} = \frac{3}{4} = 0.75 = 75\%$.

Likely wrong response: reports 133.3% from reversing the division.

Likely misconception: part and whole are reversed.

Next move: state which quantity is the part before dividing.

5. Discount \$18; sale price \$54. Valid methods include subtracting 25% of 72 or multiplying by 0.75.

Likely wrong response: gives \$18 as the final price.

Likely misconception: change amount confused with final amount.

Next move: label original, change, and final.

6. $120 \times 1.15 = 138$; $138 \times 0.90 = 124.20$.

Likely wrong response: subtracts 10% of the original 120.

Likely misconception: base is not updated after the increase.

Next move: circle the new base before the second percent.

7. For example, 100 becomes 120 and then 96, so the changes do not cancel.

Likely wrong response: returns to 100.

Likely misconception: equal percentages are assumed to mean equal amounts.

Next move: compute each change amount and compare them.

8. Answers vary. The explanation must identify the relevant reference whole or changing base.

Likely wrong response: describes computation only.

Likely misconception: the reference quantity remains implicit.

Next move: ask 'percent of what?'.

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Part 3 - Ratios and Rates

1. red:blue = 3:5; red/total = $\frac{3}{8}$ and blue/total = $\frac{5}{8}$.

Likely wrong response: uses denominators of 5 for part-to-whole fractions.

Likely misconception: part-to-part and part-to-whole are confused.

Next move: build the total $3 + 5$ before writing fractions.

2. 12:20. From 3:5 to 6:10 the factor is 2; from 6:10 to 12:20 the factor is 2.

Likely wrong response: writes 12:15.

Likely misconception: only one term is scaled.

Next move: apply the same factor to both terms.

3. Equivalent. Both simplify to 2:3; cross-products also agree: $8 \times 21 = 12 \times 14 = 168$.

Likely wrong response: compares differences.

Likely misconception: ratio equivalence treated additively.

Next move: require either simplest form or cross-product evidence.

4. 70 km/h; in 5.5 hours the car travels 385 km.

Likely wrong response: reports 38.18 km/h or 75.5 km.

Likely misconception: division direction or rate application is wrong.

Next move: track km/h and multiply hours by km/h.

5. Store A is \$2.50 per item; Store B is \$2.40 per item, so Store B is the lower unit price.

Likely wrong response: chooses Store A because \$15 is lower.

Likely misconception: totals compared across unequal quantities.

Next move: normalize both offers to one item.

6. A lower total may correspond to a smaller package. A fair comparison requires a common unit.

Likely wrong response: states only that package sizes differ.

Likely misconception: comparison rule is incomplete.

Next move: name the common unit and compute it.

7. Speed is 210 km divided by 3 h = 70 km/h, not 3 h divided by 210 km.

Likely wrong response: keeps h/km as the requested speed.

Likely misconception: units do not match the requested rate.

Next move: write the requested unit before choosing division order.

8. Answers vary. A ratio table is useful when several equivalent pairs or scale factors must be visible.

Likely wrong response: says it is always better.

Likely misconception: method choice is not tied to purpose.

Next move: ask what additional relationship the table reveals.

Recalculation decision

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Part 4 - Proportions, Graphs, Scale, and Mixed Applications

1. $x = 15$. Scale factor: 5 to 25 is $\times 5$, so $3 \times 5 = 15$. Cross-product check: $5x = 75$.

Likely wrong response: uses $x = 8$ or $x = 12$.

Likely misconception: additive change replaces multiplicative equivalence.

Next move: mark corresponding numerator and denominator positions.

2. y-values are 0, 5, 15, and 25. The 2.5 means 2.5 output units for every 1 input unit.

Likely wrong response: adds 2.5 successively despite uneven x-values.

Likely misconception: table entries treated as a running sequence.

Next move: calculate $y = 2.5x$ independently for each row.

3. It is not directly proportional because a nonzero y-intercept means y is not 0 when x is 0.

Likely wrong response: calls every straight line proportional.

Likely misconception: constant slope confused with direct proportion.

Next move: apply the origin test.

4. 7.5 cm represents 90 km; 54 km represents 4.5 cm.

Likely wrong response: uses the same operation in both directions.

Likely misconception: scale direction is not tracked.

Next move: write cm-to-km and km-to-cm arrows.

5. Similar. Both corresponding dimensions scale by 2.5: 4 to 10 and 7 to 17.5.

Likely wrong response: checks only one side pair.

Likely misconception: one matching ratio is treated as sufficient.

Next move: verify every corresponding pair.

6. Multi-step percent: $\$160 \times 0.75 \times 1.08 = \$129.60$.

Likely wrong response: \$134.40 from applying tax to the original price.

Likely misconception: the tax base is not updated.

Next move: label original, sale price, and taxed price.

7. Only the 6 cm map length is known. No real distance can be calculated without the scale.

Likely wrong response: invents a scale or reports 6 km.

Likely misconception: missing information is ignored.

Next move: list known quantities and the missing conversion relationship.

8. Answers vary. The second representation must independently confirm the relationship or result.

Likely wrong response: redraws the same work without a check.

Likely misconception: representation is decorative rather than evidentiary.

Next move: ask what new evidence the second representation supplies.

Recalculation decision

Route to the narrowest confirmed gap. Record whether the evidence points to relationship classification, representation, operation, computation, units or base, interpretation, or checking. Do not route from one isolated miss.